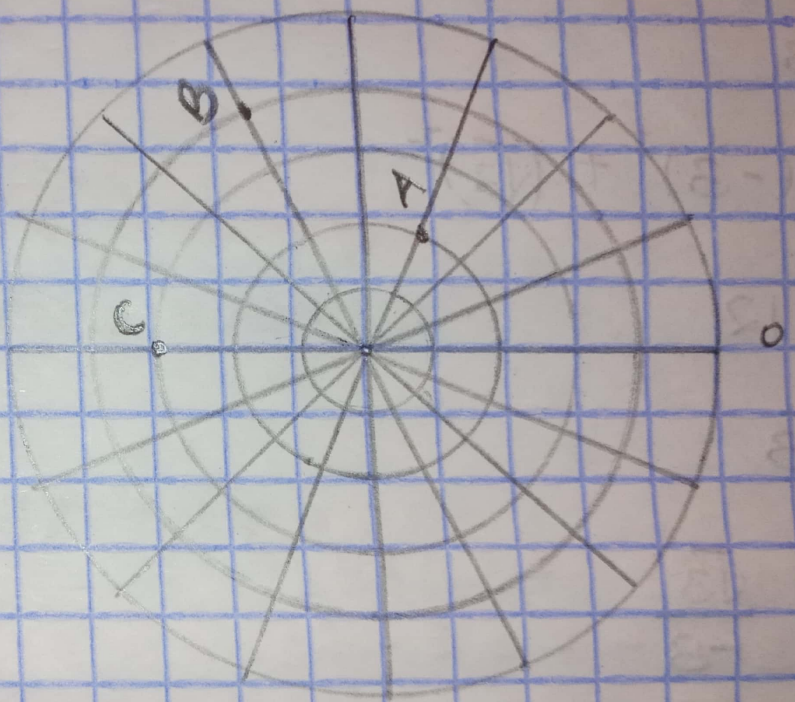
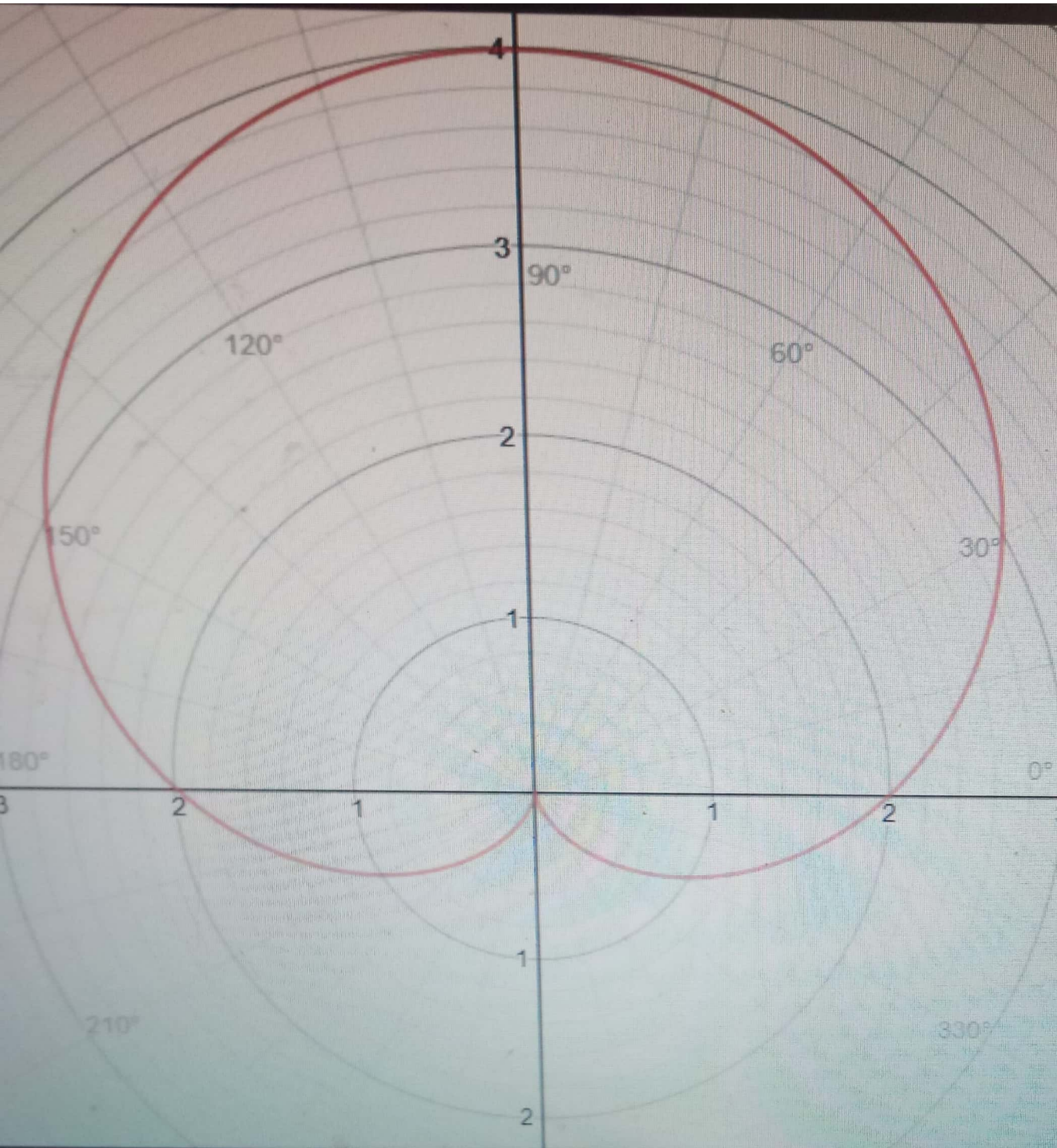




2. $r = 2 + 2 \sin \theta$

r	θ
2	0
3	$\pi/6$
$2 + \sqrt{2}$	$\pi/4$
$2 + \sqrt{3}$	$\pi/3$
4	$\pi/2$
$2 + \sqrt{2}$	$3\pi/4$
2	π
1	$7\pi/6$
$2 - \sqrt{2}$	$5\pi/4$
$2 - \sqrt{3}$	$4\pi/3$
0	$3\pi/2$
$2 - \sqrt{2}$	$7\pi/4$
2	2π



$$3(a) (-3, \sqrt{3})$$

$$x = -3, y = \sqrt{3}$$

$$r^2 = x^2 + y^2 = (-3)^2 + (\sqrt{3})^2$$

$$r^2 = 9 + 3 = 12$$

$$r = \sqrt{12} = 2\sqrt{3}$$

$$\tan \theta = \frac{y}{x} = \frac{\sqrt{3}}{-3}$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{3}}{-3} \right) = -30^\circ$$

$$= 180 - 30 = 150^\circ$$

$$= (2\sqrt{3}, 150^\circ) = (2\sqrt{3}, \frac{5}{6}\pi)$$

$$(b) (-\sqrt{3}, \frac{2\pi}{3})$$

$$r = -\sqrt{3}, \theta = \frac{2\pi}{3}$$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$x = -\sqrt{3} \cos \left(\frac{2\pi}{3} \right) \quad y = -\sqrt{3} \sin \left(\frac{2\pi}{3} \right)$$

$$x = -1$$

$$y = -\frac{\sqrt{3}}{2}$$

$$x = \frac{\sqrt{3}}{2}$$

$$y = -\frac{3}{2}$$

$$= \left(\frac{\sqrt{3}}{2}, -\frac{3}{2} \right)$$

$$(c) \sqrt{3} + i$$

$$x = \sqrt{3} \quad y = 1$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\tan \theta = \frac{y}{x} = \frac{1}{\sqrt{3}} \quad \theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ$$

$$= r(\cos \theta + i \sin \theta)$$

$$= 2(\cos 30^\circ + i \sin 30^\circ)$$

$$4 \quad Z = 4(\cos 27^\circ + i \sin 27^\circ)$$

$$Z^4$$

$$(a) Z^n = r^n (\cos(n\theta) + i \sin(n\theta))$$

$$Z^4 = r^4 (\cos(4\theta) + i \sin(4\theta))$$

$$= 4^4 (\cos(4 \times 27^\circ) + i \sin(4 \times 27^\circ))$$

$$= 256 (\cos 108^\circ + i \sin 108^\circ)$$

$$(b) \sqrt[n]{Z} = \sqrt[n]{r} \left[\cos \left(\frac{\theta + 2\pi k}{n} \right) + i \sin \left(\frac{\theta + 2\pi k}{n} \right) \right]$$

$$k = 0, 1, 2, \dots, n-1$$

$$= \sqrt[3]{4} \left[\cos \left(\frac{27^\circ + 2\pi k}{3} \right) + i \sin \left(\frac{27^\circ + 2\pi k}{3} \right) \right]$$

$$k = 0, 1, 2$$

$$k = 0$$

$$\Rightarrow \sqrt[3]{4} \left[\cos \left(\frac{27^\circ}{3} \right) + i \sin \left(\frac{27^\circ}{3} \right) \right]$$

$$k = 1$$

$$\Rightarrow \sqrt[3]{4} \left[\cos \left(\frac{27^\circ + 2\pi}{3} \right) + i \sin \left(\frac{27^\circ + 2\pi}{3} \right) \right]$$

$$k = 2$$

$$\Rightarrow \sqrt[3]{4} \left[\cos \left(\frac{27^\circ + 4\pi}{3} \right) + i \sin \left(\frac{27^\circ + 4\pi}{3} \right) \right]$$

$$5 \text{ (a) Airplane: } 500(\cos 65^\circ i + \sin 65^\circ j) \\ = 211.31i + 453.15j$$

$$\text{Wind: } 100(\cos 5^\circ i + \sin 5^\circ j) \\ = 99.62i + 8.72j$$

$$= 211.31i + 453.15j + 99.62i + 8.72j$$

$$= 310.93i + 461.87j$$

$$\theta = \tan^{-1} \left(\frac{461.87}{310.93} \right) = 56.05^\circ$$

$$= N 33.95^\circ E$$

$$= \sqrt{310.93^2 + 461.87^2}$$

$$= 556.78 \text{ mph}$$

$$(b) \text{proj}_V U = \frac{U \cdot V}{\|V\|^2} \cdot V$$

$$U \cdot V = \langle 211.31, 453.15 \rangle \cdot \langle 99.62, 8.72 \rangle$$

$$= 25002.1702$$

$$\|V\|^2 = (\sqrt{99.62^2 + 8.72^2})^2 = 10000.1828$$

$$= \frac{25002.1702}{10000.1828} (99.62i + 8.72j)$$

$$= 249.07i + 21.8j$$

$$6. \quad u = \langle 1, -4 \rangle \quad v = \langle -3, 2 \rangle$$

(a) $3u$

$$3\langle 1, -4 \rangle \\ = \langle 3, -12 \rangle$$

(b) $u - 4v$

$$4v = 4\langle -3, 2 \rangle = \langle -12, 8 \rangle \\ = \langle 1, -4 \rangle - \langle -12, 8 \rangle \\ = \langle 13, -12 \rangle$$

(c) $u \cdot v$

$$= \langle 1, -4 \rangle \cdot \langle -3, 2 \rangle \\ = (1 \times -3) + (-4 \times 2) = -3 - 8 \\ = -11$$

(d) $|u| = \sqrt{1^2 + (-4)^2} = \sqrt{1 + 16} \\ = \sqrt{17}$

(e) Angle between u and v

$$\cos \theta = \frac{u \cdot v}{\|u\| \cdot \|v\|}$$

$$u \cdot v = -11 \quad \|v\| = \sqrt{(-3)^2 + 2^2} = \sqrt{9 + 4}$$

$$\|u\| = \sqrt{17} \quad = \sqrt{13}$$

$$\cos \theta = \frac{-11}{\sqrt{17} \cdot \sqrt{13}} = \frac{-11}{121}$$

$$\theta = \cos^{-1} \left(\frac{-11}{221} \right) = 92.85^\circ$$

$$7 \quad x^2 + 4x + 4y^2 - 8y + 4 = 0$$

(a) Ellipse

~~$$(b) \quad x^2 + 4x + 4 + 4y^2 - 8y + 16 + 4 - 4 - 16 = 0$$~~

~~$$(x+2)^2 +$$~~

~~$$(b) \quad x^2 + 4x + 4y^2 - 8y = -4$$~~

~~$$(x^2 + 4x) + 4(y^2 - 2y) = -4$$~~

~~$$\frac{1}{4}(x^2 + 4x) + (y^2 - 2y) = -1$$~~

~~$$\frac{1}{4}(x^2 + 4x + 4) + (y^2 - 2y + 1) = -1 + \frac{1}{4}(4) + 1$$~~

~~$$\frac{1}{4}(x+2)^2 + (y-1)^2 = 1$$~~

~~$$\text{Centre } (h, k) = (-2, 1)$$~~

~~$$\text{Vertices} = (h+a, k), (h-a, k)$$~~

~~$$= (-2+2, 1), (-2-2, 1)$$~~

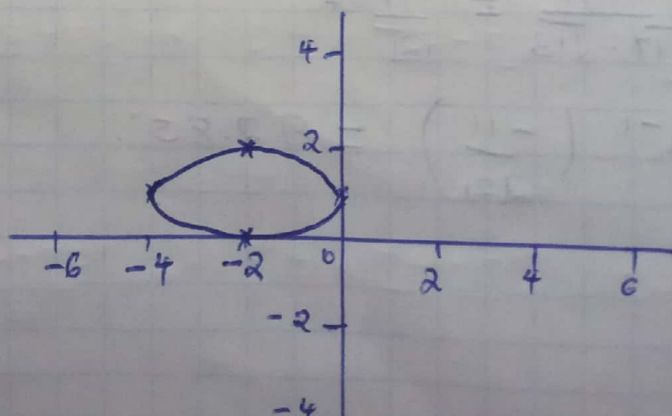
~~$$= (0, 1), (-4, 1)$$~~

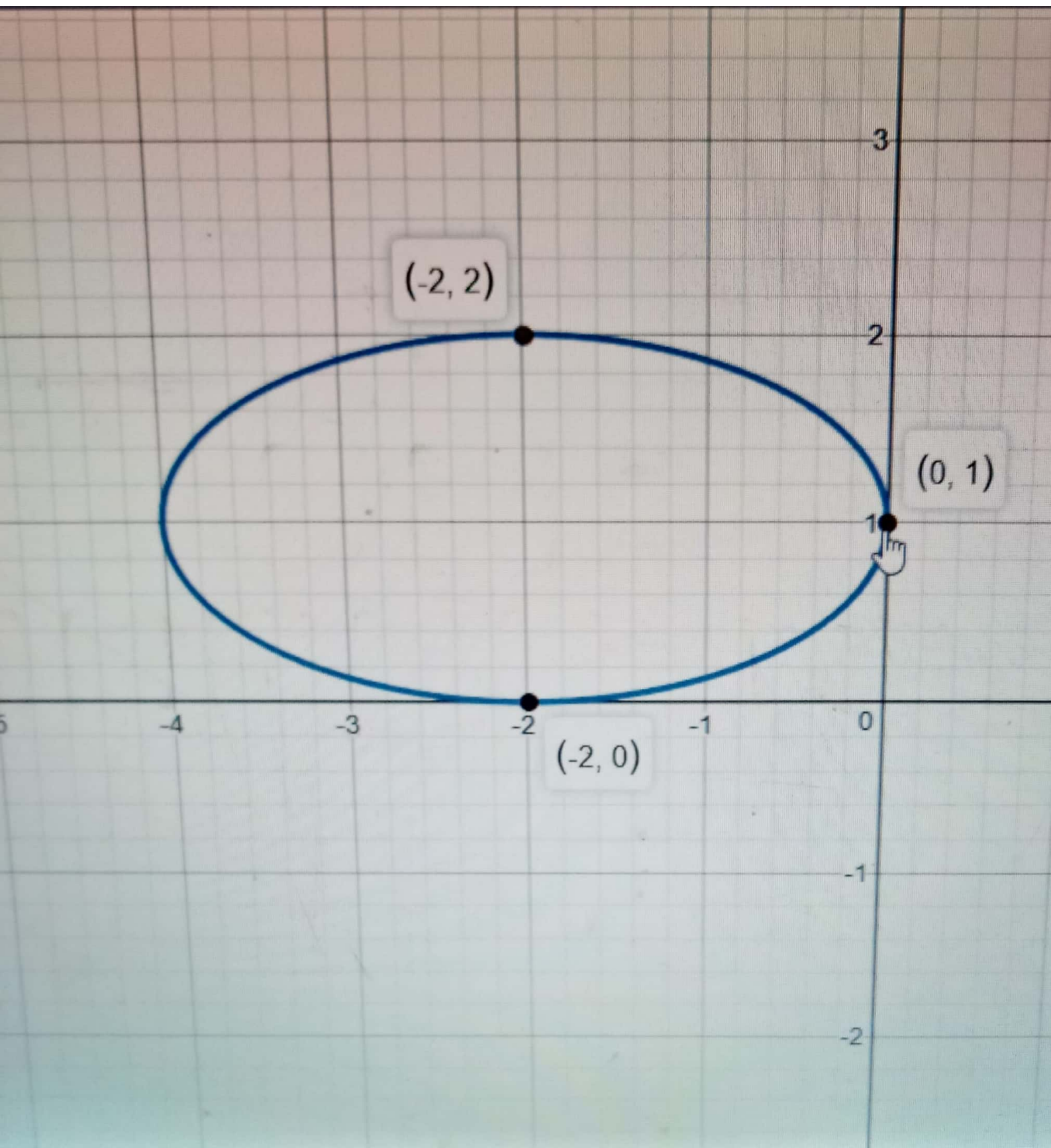
~~$$c = \sqrt{a^2 - b^2} = \sqrt{4-1} = \sqrt{3}$$~~

~~$$\text{Foci} = (h+c, k), (h-c, k)$$~~

~~$$= (-2+\sqrt{3}, 1), (-2-\sqrt{3}, 1)$$~~

(c)





8 (a) Hyperbola

$$(b) \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2}$$

$$\frac{(y+1)^2}{9} - \frac{(x-3)^2}{4} = 1$$

$$\text{Centre } (h, k) = (3, -1) \quad a = 3, \quad b = 2$$

$$\text{Vertices} = (h, k+a), (h, k-a)$$

$$= (3, -1+3), (3, -1-3)$$

$$= (3, 2), (3, -4)$$

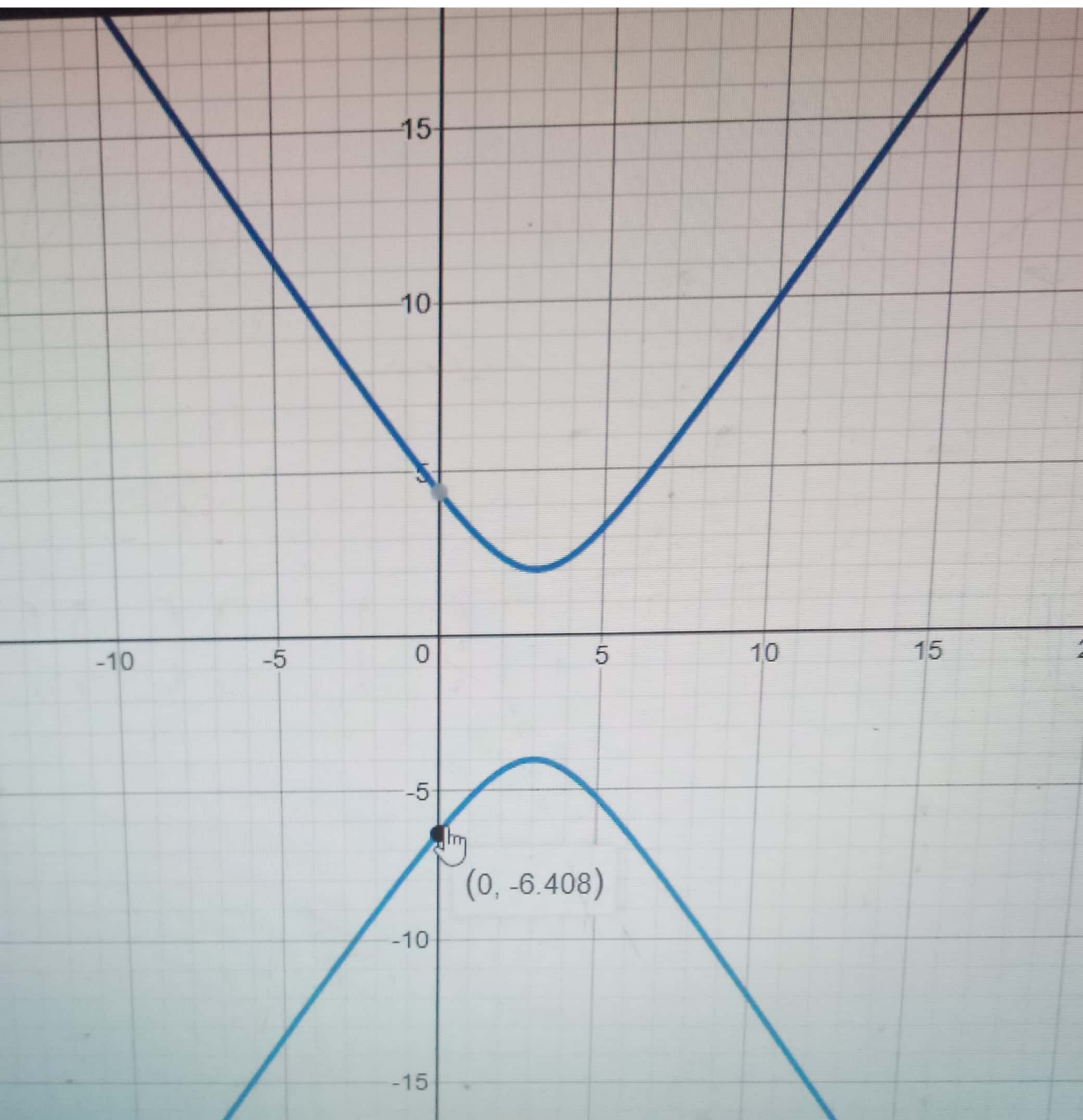
Asymptotes

$$y = \pm \frac{a}{b} (x-h) + k$$

$$= \pm \frac{3}{2} (x-3) - 1$$

$$= \pm \left(\frac{3}{2}x - \frac{9}{2} - 1 \right) = \pm \left(\frac{3}{2}x - \frac{11}{2} \right)$$

$$y = \frac{3}{2}x - \frac{11}{2}, \quad y = -\frac{3}{2}x + \frac{11}{2}$$



9. Foci = $(7, 3), (7, 7)$

Vertex = $(7, 6)$

Vertices = $(h, k+a), (h, k-a)$

foci = $(h, k+c), (h, k-c)$

$$\begin{array}{r} k+c=7 \\ -k-c=3 \\ \hline 2c=4 \end{array}$$

$$c=2$$

$$\begin{array}{r} k+2=7 \\ k=5 \end{array}$$

$$\text{Vertex} = (7, 6) = (h, k+a)$$

$$k+a=6$$

$$\begin{array}{r} 5+a=6 \\ a=1 \end{array}$$

$$b^2 = c^2 - a^2 = 4 - 1 = 3$$

$$b = \sqrt{3}$$

$$\text{Equation: } \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$= \frac{(y-5)^2}{4} - \frac{(x-7)^2}{3} = 1$$

